Algebra 2 Regents Exam Overview

The Algebra 2 Regents Exam is structured into four key units:

- **Number and Quantity**: 5-12%
- **Algebra**: 35-44%
- **Functions**: 30-40%
- **Statistics and Probability**: 14-21%

The exam is 3 hours long and comprises 37 questions across four parts:

Part	Format	Points per Question	Total Points
Part 1	Multiple Choice	2	48
Part 2	Short Answer	2	16
Part 3	Multi-Part	4	16
Part 4	Graph Drawing	6	6
	Total Raw Score		**86**

The Region's exam is positively scaled, so your scaled score will be higher than your raw score.

- A raw score of 66/86 (77%) is needed for a Level 5.
- A raw score of 47/86 is needed for a Level 4.
- A raw score of 20/86 (30%) is needed to pass.

Unit 1: Number and Quantity

Number Systems

The **real number system** includes:

- **Rational Numbers**: Decimals that terminate or repeat.
  - **Integers**: Numbers with only zeros after the decimal point (e.g., -3.0).
  - **Whole Numbers**: Zero or positive numbers.
  - **Natural/Counting Numbers**: Numbers you can count on your fingers (one and greater).
- **Irrational Numbers**: Decimals that do not repeat or terminate (e.g., π, e).

Any natural number is a whole number, an integer, and a rational number. However, a whole number is not necessarily a natural number.

Number Properties

Key properties of radicals, rational exponents, logs, and exponents will be needed for the exam.

Polynomial Division

Polynomial division is similar to long division, but uses polynomials. For example, dividing $2x^2 + 7x + 6$ by $x + 2$:

1. Determine what to multiply $x + 2$ by to cancel out the first term ($2x^2$). Multiply $x + 2$ by $2x$ to get $2x^2 + 4x$.
2. Subtract this from the original polynomial:

   $$(2x^2 + 7x + 6) - (2x^2 + 4x) = 3x + 6$$

3. Multiply $x + 2$ by 3 to get $3x + 6$.
4. Subtract this from $3x + 6$:

   $$(3x + 6) - (3x + 6) = 0$$

In this case, since there is no remainder, $x + 2$ is a **factor** of the polynomial according to the **Factor Theorem**. This is written as $x - A$, so $x - (-2)$ is a factor.

If there is a remainder, the **Remainder Theorem** applies. The remainder will be $\frac{F(a)}{\text{polynomial dividing}}$

Note: Synthetic division is allowed but not required.

Rational Functions

Rational functions are like fractions with polynomials in the numerator and denominator. All fraction rules apply.

Rationalizing Denominators

To rationalize a denominator with a square root, multiply the numerator and denominator by the conjugate of the denominator.

For example, if you have $\frac{A}{B-\sqrt{C}}$, multiply by $\frac{B+\sqrt{C}}{B+\sqrt{C}}$:

$$\frac{A}{B-\sqrt{C}} \times \frac{B+\sqrt{C}}{B+\sqrt{C}} = \frac{A(B+\sqrt{C})}{B^2-C}$$

Factoring

1. Greatest Common Factor: Factor out the greatest common factor.
    - Example: $5x - 3x^2 = x(5 - 3x)$
2. Difference of Squares: If you have a square number minus another square number:

    $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$

    In general: $a^2 - b^2 = (a + b)(a - b)$

Factoring Polynomials

Factoring with Poly Roots Calculator

If other methods fail, use the poly roots calculator to find roots. Remember to factor out any greatest common factors before using the calculator.

Example: Factoring $x^2 - 4x + 3$

Find two numbers that:

- Add up to -4
- Multiply to 3

The factors of 3 are:

- 1 and 3
- -1 and -3

Since -1 + (-3) = -4, the factors are (x - 1) and (x - 3). Therefore, the factored form of the expression is $(x-1)(x-3)$.

Check: Box multiplication of $(x-1)(x-3)$ should yield x^2-4x+3.

Factoring by Grouping

- Useful for polynomials with a degree of 3 or greater.

Steps:

1. Make two groups with a greatest common factor.
2. Factor out the greatest common factor.
3. Simplify.

Example: Factor x^3+2x^2-3x-6

1. Group the terms: $(x^3+2x^2)+(-3x-6)$
2. Factor out x^2 from the first group: $x^2(x+2)$
3. Factor out -3 from the second group: $-3(x+2)$
4. Rewrite the expression: $x^2(x+2)-3(x+2)$

If factoring by grouping is possible, the terms in the parentheses must be the same.

5. Factor out the common term $(x+2)$: $(x^2-3)(x+2)$

Thus, the factored form is $(x^2-3)(x+2)$.

Complex Numbers

Basics of Complex Numbers

- The imaginary unit i is the basis of the complex number system.
- $i=\sqrt{-1}$
- $i^2=-1$

Complex Number Form: a + bi, where:

- a and b are real numbers.
- b is the coefficient of i.

Rationalizing Complex Numbers

Rationalizing complex numbers is similar to rationalizing normal numbers:

- Multiply the top and bottom of the fraction by the **conjugate**.
- The conjugate of c + di is c - di.

If you have a rational function in complex form, multiply both the numerator and the denominator by the conjugate.

Functions

Overview

Topics covered:

- Domain and range
- Composition functions
- Inverse functions
- One-to-one and onto functions
- End behavior
- Multiplicity
- Transformations
- Applications
- Logarithms
- Regression functions

Defining a Function

Functions must pass the **vertical line test**: a vertical line should only intersect the function once for each x-value. A function should only have one y-value for each x-value.

Domain

- The set of x-values for which the function produces an output.
- For rational functions, the domain is all real numbers except where the denominator equals zero.

Example: $f(x) = \frac{x-4}{x+2}$

- The denominator is zero when x = -2.
- Domain: all real numbers where $x \neq -2$

Radical Functions:

- For functions like square root or cube root, the domain is all real numbers except when the radicand is less than zero.

Example: $f(x) = \sqrt{x-5}$

- $x - 5 = 0$ when $x = 5$
- $x - 5 < 0$ for all $x < 5$
- Domain: all real numbers greater than or equal to 5 ($x \geq 5$).

Range

- The set of all y-values that the function is defined for.
- Typically found graphically rather than algebraically.

One-to-One and Onto Functions

- One-to-One Functions: No repeating x or y values.
  - Must pass the horizontal line test (in addition to the vertical line test).
- Onto Functions: All x-values have a defined y-value.

Composition Functions

A function inside of a function, written as $f(g(x))$. Denoted as $f \circ g(x)$, which means $f(g(x))$.

Example: Given $g(x) = 3x^2$ and $f(x) = \sqrt{9x}$, find $h(x) = f(g(x))$.

$h(x) = f(g(x)) = \sqrt{9 \times 3x^2} = \sqrt{27x^2}$

Inverse Functions

Obtained by reflecting a one-to-one function over the line $y = x$.

To find the inverse function:

1. Swap the x and y values in the original function.
2. Solve for y.

Example: Given $y = 3x^2 + 5$, find the inverse function.

1. Swap x and y: $x = 3y^2 + 5$
2. Solve for y:

$x - 5 = 3y^2$

$\frac{x-5}{3} = y^2$

$y = \sqrt{\frac{x-5}{3}}$

Notation: $f^{-1}(x)$ denotes the inverse function of f(x).

End Behavior and Multiplicity

End behavior is pretty simple and on the test if you...

Polynomial End Behavior

If you don't know the end behavior of a polynomial, you can graph it. At the ends, the functions will either go to positive or negative infinity.

Here's a quick trick:

- If the **leading degree** of your function is **odd** (like x^3 or x^5), the ends will point in **different directions**.
- If the **leading degree** is **even** (like x^4 or x^6), they'll point in the **same direction**.

If the coefficient of the leading degree is positive (like $4x^3$) for odd functions, it will look like $y = x$:

- As x approaches positive infinity, y approaches positive infinity.
- As x approaches negative infinity, y approaches negative infinity.

If it's negative (like $-3x^5$) for odd functions, it'll look something like the reverse of $y = x$.

Even functions will have both ends going up for a positive leading coefficient and both ends going down for a negative one.

Multiplicity

Multiplicity is how many times something bounces. As you go from a multiplicity of 1 to 3 to 5, the graph stretches a little more. Similarly, going from 2 to 4 to 6 also increases the stretch.

Transformations

There are six types of transformations:

1. Horizontal Translations (Horizontal Shifts)
2. Horizontal Dilations (Horizontal Shrinking or Scaling)
3. Reflections over the y-axis
4. Reflections over the x-axis
5. Vertical Dilations
6. Vertical Translations

Let's start with horizontal translations and y-axis reflections first.

Horizontal Translations

The equation for a horizontal translation involves taking $f(x)$ and transforming it into $f(x \pm a)$.

For example, if we had x^2, a horizontal translation would be $(x \pm 4)^2$.

Subtracting `$a$` (i.e., $f(x - a)$) results in a **right shift**. Each point is shifted by the value of `$a$`.

Note that the change is happening **within the parentheses**. Adding a results in a **leftward shift**, while subtracting a results in a **rightward shift**.

Horizontal Dilations

Dilations shrink or stretch functions. If we multiply by some integer K, it'll get skinnier, and if we multiply by some $\frac{1}{K}$, it'll get smaller. This occurs within the parentheses, as we see by our rule here: $f(K \times x)$ or $f(\frac{1}{K} \times x)$.

Y-Axis Reflection

For a y-axis reflection, we take the original function and flip it over the y-axis. The equation for that is $f(x)$ becomes $f(-x)$. Again, the change is occurring inside the parentheses.

Vertical Translations

For a vertical translation, adding a moves it up, and subtracting a moves it down. This is different from horizontal translations because the change is **outside the parenthesis**. For example, it's not going to become like $(x-4)^2$; it'll be $x^2 - 4$.

Vertical Dilations

Vertical dilations are essentially the same, except again, the rule changes; it's outside the parentheses, so it's $K \times f(x)$ and $\frac{1}{K} \times f(x)$.

X-Axis Reflection

X-axis reflection is also similar. $f(x)$ becomes $-f(x)$. Again, outside of the parenthesis, and we're going to flip over the x-axis.

Acronyms for Remembering Transformations

- HIYA: Horizontal Inside Y-Axis. Any transformation involving the y-axis (like reflection and horizontal dilation and translations) involves changing the rule on the inside of the parentheses.
- VXV: Vertical X-Axis Vertical Outside. All these rules are going to happen outside of the parenthesis.

Remember:

- Translations always involve adding or subtracting an `$a$`.
- Dilations involve multiplying by `$K$` or `$\frac{1}{K}$`.
- Reflections involve adding in a negative.

Order of Transformations

The order matters! You always must perform your transformations in the following order: HD RV (Horizontal Translations, Dilations, Reflections, and Vertical translations). If you don't do this, it'll mess up how your image is constructed. Remember this by thinking "Helicopters Do Rise Vertically."

Even and Odd Functions

- A function is called even if it is symmetric about the Y-axis (e.g., the cosine function or the function x^2).
- A function is called odd if it has 180-degree rotational symmetry about the origin, so $f(-x) = -f(x)$ for all `$x$` in the domain (e.g., x^3 or $\sin(x)$).

Regression

Know how to recognize these models and be able to use your calculator to find regression equations when you're given those data points:

- Linear
- Quadratic
- Cubic
- Exponential
- Logarithmic

Applications of Logs ▯

Here's the basic way to write a logarithmic equation:

$y = \log_b(x)$

Here are some ways that it's actually useful and applicable in real life:

Exponential Growth and Decay

If we have this equation:

$A(t) = P \times (1 \pm r)^T$

We can draw exponential growth or decay:

- P: Initial value/start value
- 1: Represents 100%
- r: Rate (expressed as a decimal, e.g., 3% interest = 0.03)
- T: Time

Compound Interest

Compound interest can also be shown using this equation:

$A = P(1 + \frac{r}{n})^{nT}$

The numbers mean the same thing, and n is the number of compounds per year.

Compounding Period	n
Monthly	12
Quarterly	4
Daily	365
Yearly	1

Continuous Compounding

Continuous compounding requires the use of the number e (approximately equal to 2.72) and is the limit of the function $(1 + \frac{1}{n})^n$

Continuous Compounding

For continuous compounding, where n approaches infinity, use the following equation:

$A = Pe^{rt}$

Where:

- A = the future value of the investment/loan, including interest
- P = the principal investment amount (the initial deposit or loan amount)
- r = the annual interest rate (as a decimal)
- t = the number of years the money is invested or borrowed for
- e = Euler's number (approximately equal to 2.71828)

Trigonometric Functions

Overview of trigonometric functions:

- Radians and degrees
- Trig functions
- The unit circle
- Special triangles
- Identities
- Inverses
- Graphs and parts of trig functions

Radians and Degrees Conversion

- Degrees to radians: Multiply by $\frac{\pi}{180}$
- Radians to degrees: Multiply by $\frac{180}{\pi}$

Primary Trig Functions and Their Reciprocals

Trig Function	Definition	Reciprocal	Definition of Reciprocal
$\sin(\theta)$	$\frac{y}{r}$	$\csc(\theta)$	$\frac{r}{y}$
$\cos(\theta)$	$\frac{x}{r}$	$\sec(\theta)$	$\frac{r}{x}$
$\tan(\theta)$	$\frac{y}{x}$	$\cot(\theta)$	$\frac{x}{y}$

θ is theta.

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- $\sin(\theta)$ = Opposite / Hypotenuse
- $\cos(\theta)$ = Adjacent / Hypotenuse
- $\tan(\theta)$ = Opposite / Adjacent

Understanding Triangle Sides relative to an Angle

- **Hypotenuse**: The longest side of a right triangle.
- **Adjacent**: The side next to the angle (that is not the hypotenuse).
- **Opposite**: The side opposite the angle.

Special Triangles

Two special triangles:

- 30-60-90 triangle (ratio $1 : \sqrt{3} : 2$)
- 45-45-90 triangle (ratio $1 : 1 : \sqrt{2}$)

Example: sin(30°)

$\sin(30°) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2}$

Example: sin(45°)

$\sin(45°) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ (after rationalizing the denominator)

Important Note

It's important to memorize these special triangles to provide exact values (with square roots) when asked, as calculators typically provide decimal approximations.

Trig Identities

Reciprocal Identities

- $\sin(\theta) = \frac{1}{\csc(\theta)}$
- $\cos(\theta) = \frac{1}{\sec(\theta)}$
- $\tan(\theta) = \frac{1}{\cot(\theta)}$

Tangent and Cotangent Identities

- $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$

Pythagorean Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$

The Unit Circle

A circle with a radius of 1.

- The radius serves as the hypotenuse of a triangle formed by the x-axis, y-axis, and the radius itself.
- Coordinates on the unit circle are (x, y), where x corresponds to the cosine of the angle and y corresponds to the sine of the angle.

Using the Unit Circle

To find sin(30°) or $\sin(\frac{\pi}{6})$:

1. Locate the 30° angle on the unit circle.
2. Identify the y-value of the point where the angle intersects the circle.
3. $\sin(30°) = \frac{y}{r} = \frac{0.5}{1} = \frac{1}{2}$

Inverse Trigonometric Functions

Used when you know the sine, cosine, or tangent value but need to find the angle. Examples: arcsin, arccos, arctan.

Inverse Trig Functions

When $x = \frac{1}{2}$, to find the angle, use the inverse trig function:

$arcsin(\frac{1}{2}) = x$

- This gives $x = 30$ degrees, if your calculator is in degree mode.
- In radian mode, the result is $x = \frac{\pi}{6}$.

Inverse trig functions have **restricted domains** and **ranges** to ensure they are **one-to-one functions**.

When finding the $arcsin$ or $arctan$ of a value, you might need to adjust the angle to find the correct quadrant. If you find an angle in one quadrant and need the corresponding angle in another:

- Add 90 degrees or subtract from 180 degrees, depending on the quadrants.

General Trig Equation

The general form of a trig equation is:

$y = A \times sin(B_2 \times (x - C)) + D$

Where:

- A = **Amplitude** = $\frac{1}{2} \times |max\ value - min\ value|$
- B_1 = Trig function (sin, cos, tan, etc.)
- B_2 = **Frequency**. The period is $\frac{2\pi}{frequency}$.
- C = **Horizontal shift** (positive is right, negative is left)
- D = **Vertical shift** (positive is up, negative is down)

Identifying a Trig Function

To identify a trig function and determine its equation from a graph:

1. **Identify the type of function**:
   - Sine function starts at zero and goes up.
   - Cosine function starts at its maximum value.
   - Tangent function has a distinct shape.
2. **Amplitude**:
   - Find the max and min values.
   - Amplitude is half the difference between the max and min.
3. **Frequency**:
   - Count how many cycles are completed in 2π.
4. **Horizontal Shift**:
   - Determine how much the function is shifted left or right compared to its standard position.
5. **Vertical Shift**:
   - Find the midline (average of max and min values).
   - The vertical shift is how much the midline is above or below the x-axis.

Example

Given a sine function:

- Amplitude: 3
- Frequency: 2
- Horizontal Shift: $\frac{\pi}{2}$ to the right
- Vertical Shift: 5 units up

The equation is:

$y = 3 \times sin(2 \times (x - \frac{\pi}{2})) + 5$

Linear Equations

Linear equations are functions with a **degree of one** and form **straight lines** when graphed. They have a clear slope.

- **Slope** is the average rate of change = $\frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$
  - Given two points $(x_a, f(x_a))$ and $(x_b, f(x_b))$, the slope is:

  $m = \frac{f(x_b) - f(x_a)}{x_b - x_a}$

Examples of Slopes

Slope Type	Description
Positive	Line goes up from left to right
Negative	Line goes down from left to right
Zero	Horizontal line
Undefined	Vertical line

Example Problem

Given two points (3, 6) and (0, 4):

- Change in y = +2
- Change in x = +3

Slope, $m = \frac{2}{3}$

Forms of Linear Equations

- **Slope-intercept form**:

  $$y = mx + b$$

  - m is the slope.
  - b is the y-intercept.

- **Point-slope form**:

  $$y - y_1 = m(x - x_1)$$

  - m is the slope.
  - (x_1, y_1) is a point on the line.

Example

Using the point (3, 6) and slope $\frac{2}{3}$, the point-slope form is:

$$y - 6 = \frac{2}{3}(x - 3)$$

Three-Variable Linear Systems

For multiple-choice questions, use the "LinSolve" function on your calculator.

For short-answer questions, show your work using the following steps:

1. **Group equations**:
    - Group the first and second equations together.
    - Group the second and third equations together.
2. **Eliminate a variable**:
    - Eliminate the same variable from both systems.
3. **Consolidate**:
    - Combine both systems into one equation.
4. **New system**:
    - Use the new equations to create a new system.
5. **Eliminate another variable**:
    - Solve for the remaining variable.
6. **Solve**:
    - Back-substitute to find the values of the other two variables.

Example

Given the system:

1. $x + y + z = 1$
2. $2x + \dots$

Solving Systems of Equations

Here's how to solve a system of three equations with three variables:

1. Split the equations into two groups.
    - G1: $4y + 6z = 2x \; 3y - 5z = 11$
    - G2: Use the equations from G1.
2. Eliminate the same variable from both systems.
    - Multiply the top equation by -2 to cancel out the x's: $2y + 4z = 0$
    - Multiply the second equation by 2 to cancel out the x's: $10y - 4z = 24$
3. Create a new system of equations using the two new equations:
    - $2y + 4z = 0$
    - $10y - 4z = 24$
4. Solve for one of the variables.
    - Eliminate z by adding the equations together.
    - Solve for y: $y = 2$
5. Substitute the value of y back into one of the equations to solve for z:
    - $z = -1$
6. Substitute the values of y and z into one of the original three-variable equations to solve for x:
    - $x = 0$

Therefore, the final solution is: $y = 2$, $z = -1$, and $x = 0$.

Quadratics

- Definition: Polynomials with a degree of two.
- Shape: U-shaped or horseshoe-shaped.

Standard Form

The standard form of a quadratic equation is:

$y = ax^2 + bx + c$

Properties

- Symmetrical
- Axis of Symmetry: $x = -\frac{b}{2a}$
  - b and a are the same values found in the standard form equation.
- If the quadratic is not in standard form where a, b, and c are all integers, manipulate it algebraically to get it into standard form.
- Sum of Roots: Illustrated by the equation using the same b and a.
- Product of Roots: Illustrated by the equation using the same b and a.
  - Roots are the x-intercepts or zeros.

Quadratic Formula

The quadratic formula is:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Given on the reference table.

Discriminant

- The discriminant is the most important part of the quadratic equation: $b^2 - 4ac$.
- It determines the number of real roots and the nature of those roots.
  - If $b^2 - 4ac < 0$: No real roots or x-intercepts (complex roots). The function never hits the x-axis and has a y-intercept that is not zero.
  - If $b^2 - 4ac = 0$: One x-intercept.
  - If $b^2 - 4ac > 0$: Two x-intercepts.
    - If $b^2 - 4ac$ is a perfect square: Rational roots.
    - If $b^2 - 4ac$ is not a perfect square: Irrational roots.

Focus and Directrix

The focus of a quadratic is a point on the line of symmetry that is the same distance from the directrix. Both are perpendicular to the line of symmetry.

- The distance from the focus to the vertex and from the vertex to the directrix is p. The distance from the focus to the directrix is $2p$.
- Occurs at the same x-value as the turning point (min or max).

Vertex Form

- Vertex form: $y = \frac{(x-h)^2}{4p} + k$
  - (h, k) is the vertex, where h is the x-value of the turning point and k is the y-value.
  - Focus: $(h, k+p)$
  - Directrix: $y = k - p$

Vertical vs. Horizontal Parabolas

Feature	Vertical Parabola	Horizontal Parabola
Orientation	Opens up or down	Opens left or right
Directrix	$y = k - p$	$x = h - p$
Focus	$(h, k+p)$	$(h+p, k)$
Vertex Form	$y = \frac{(x-h)^2}{4p} + k$	$x = \frac{(y-k)^2}{4p} + h$

Practice Problem

- From the June 2023 practice Regents exam.
- A similar problem was on the January 2024 Regents exam.

Problem: The directrix is $y = 4$. Find the equation.

1. Determine it's a **vertical parabola** because the directrix is $y = k - p$
2. Therefore, $4 = k - p$ and $k = p + 4$.
3. The point from the focus is $(h, k + p) \implies h = 0$ and $k + p = 6 \implies k = 6 - p$.
4. Set the equations equal to each other:

   $p + 4 = 6 - p$

   $2p = 2$

   $p = 1$
5. Solve for k:

   $k = 6 - 1$

   $k = 5$
6. Plug the values into the vertex form equation:

   $y = \frac{(x-0)^2}{4\times1} + 5$

   $y = \frac{x^2}{4} + 5$

Sequences and Series

- **Sequences**: Lists formed with terms.
- **Series**: Sum of the terms in a sequence.

Types of Sequences

1. **Arithmetic**: Like linear equations (add or subtract to get to the next term).
2. **Geometric**: Like exponential equations (multiply or divide by a ratio to get to the next term).

- Defined only for integers, not decimals.

Sequences

Sequences can be either **explicit** or **recursive**:

- **Explicit**: Used to find the **nth** term.
- **Recursive**: Used to find the **next** term.

Key Terms:

- **D**: Common difference between terms (amount to get from one term to the next).
- **R**: Common ratio (amount you need to multiply or divide by to get to the next term).

Sigma Notation: Uses the sum symbol to add up terms in a sequence. Calculators can be used to easily find the sum of a sequence.

For example, if we have the expression: $\sum_{i=2}^{4}(i^2+1)$

- The last term is 4.
- The first term is 2.
- The equation is i^2+1.

So, we plug in 2, 3, and 4 into the equation:
$(2^2+1)+(3^2+1)+(4^2+1)=5+10+17=32$

The final answer is 32.

Geometric Series Formula: Can be found on your reference table. Plug in the same variables as used previously to get the answer.

Statistics and Probability

This unit makes up 14-21% of exams, typically including one or two multiple-choice and two or three short-answer problems.

Types of Studies

- **Sample Surveys**:
  - Like online polls.
  - Experimenters take a randomly selected sample of answers from a larger survey and analyze them.
  - Cannot draw inferences but can draw generalizations.
- **Observational Studies**:
  - Experimenters look at something already happening and analyze the results.
  - Non-random treatment and selection.
  - Can find causation but not a greater generalization (results only represent the observed population).
  - More cost-effective.
- **Controlled Experiments**:
  - Active experimenters randomly assign different groups of people to treatment groups.
  - Participants may or may not be randomly selected.
  - Generalization can be made if participants were randomly selected; otherwise, only an inference can be concluded due to potential bias.

Blind Experiments

- **Single Blind**: Participants don't know which side of the experiment they are on (e.g., placebo vs. real treatment).
  - Eliminates some bias.
- **Double Blind**: Neither participants nor experimenters know which group is which until after the data is analyzed.
  - Eliminates the most bias.

Good experiments always involve:

- Random assignment of treatment.
- A large number of participants.
- A control group.

Core Statistical Terms and Equations

Key terms to know from Algebra 1 include **standard deviation**, **mean**, **mode**, and **median**.

Definitions of Mean and Standard Deviation:

Term	Sample	Population
Mean	$\bar{x}$	μ
Standard Deviation	s	σ

Normal Distribution Curve:

- Symmetrical.
- 68% of data is within one standard deviation from the mean.
- 95% is within two standard deviations.
- 99.7% is within three standard deviations.
- Use the `normCDF` function on your calculator to find the percentile for a specific individual within the distribution.
- In a perfectly distributed curve, the mean, mode, and median are the same value (midline of the curve).

Confidence Intervals: A measure of how good your data is.

- Formula: $CI = \bar{x} \pm Z \times \frac{\sigma}{\sqrt{n}}$
  - Where:
    - n = sample population
    - Z = value based on the confidence level

Confidence Level	Z
90%	1.645
95%	1.96
99%	2.575

Margin of Error: Greater when there are fewer people surveyed.

- Formula: $Z \times \sqrt{\frac{P\times(1-P)}{n}}$
  - Where:
    - Z = value based on confidence level (same as above)

Z-Scores: Used to find how many standard deviations a value is from the mean.

- For Samples: $Z = \frac{\text{selected value} - \bar{x}}{s}$
- For Populations: $Z = \frac{\text{selected value} - \mu}{\sigma}$

Probability

- Probability should always be written and calculated using fractions.
- $P(A \cup B)$: Probability of A or B occurring (Union).
- $P(A \cap B)$: Probability of A and B occurring (Intersection).

Probability of a Complement:

- The probability that an event (E) does not happen.
- $P(E') = 1 - P(E)$ (100% probability minus the probability the event does happen).

Mutually Exclusive Events:

- Have nothing in common and don't affect each other.
- The probability of A## Probability of Independent and Dependent Events

Independent Events

- The probability of A and B happening separately is the same as the probability of either of them happening.
- The outcome of event A does not affect the outcome of event B.
- Three ways to prove independence: Using conditional probability is the easiest.
- Conditional Probability: Denoted as $P(A|B)$, meaning the probability of A happening given that B has already happened.

Dependent Events

- The outcome of the first event affects the second event.
- Conditional Probability: It is the probability of A happening given that B has already happened, and we know the outcome of B.

Conditional Probability Formula

- Formula: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Complementary Probability

- The probability of an event not happening is $1 - P(\text{event})$.
- $P(\text{not A}) = 1 - P(A)$
- The probability of one is just 100%.
- Subtracting the probability that something will happen gives the probability that it doesn't.